

Sherman Hanna, The Ohio State University<sup>1</sup>

Planning 20 or more years before retirement involves some difficult conceptual issues. If the goal of the consumer is to maximize utility from consumption for a two year lifetime, and the utility function has constant elasticity of marginal utility with respect to consumption, then it can be shown that the optimal ratio of Year 2 consumption to Year 1 consumption is:

$$\text{Equation 1: } C_2/C_1 = ((1+R)/(1+\rho))^{(1/X)}$$

$C_1$  = Consumption in Year 1.  $C_2$  = Consumption in Year 2.  $R$  = Real interest rate.  $-X$  = elasticity of marginal utility with respect to consumption. Most estimates of  $X$  are in the range of one to six.  $\rho$  = pure rate of time preference, which might plausibly be related to the chance of death during the year, or to changes in family composition or other factors.

There is evidence that the life cycle theory does not describe household behavior (Thaler, 1990), although comprehensive alternatives in a utility maximizing framework have not been developed. Empirical tests of the life cycle theory must make restrictive assumptions to allow for mathematical manageability. White (1978) provides a relatively clear description of the model, with the assumption of a single real interest rate and a constant personal discount rate among the assumptions needed for analysis.

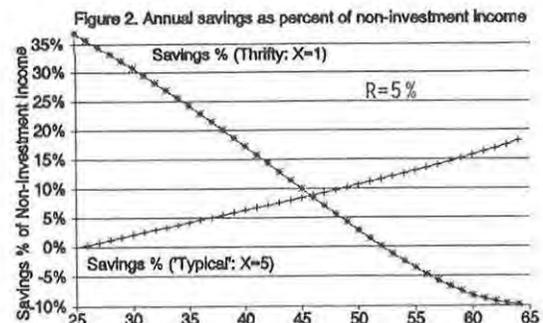
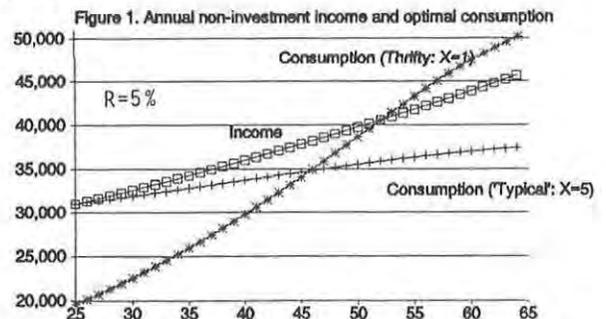
In order to provide students with insight into the life cycle theory, a computer program was written to calculate savings patterns to maximize lifetime utility, assuming Equation 1 holds for all adjacent years before retirement, borrowing is permitted at a higher real interest rate than the return on savings, and  $\rho$  is the risk of death each year for the average American of each age. It is also assumed that optimal consumption after retirement would be equal to optimal consumption for the year before retirement, and that a life annuity (with a real rate of return one percent less than the real rate of return assumed on retirement savings) would be purchased to fill the gap between optimal consumption and the pension. The program requests present age, retirement age, real pension as a percent of final salary, rate of increase in real aftertax salary, and real rate of return on retirement savings. An iterative process is used to obtain levels of consumption which maximize expected utility. The assumptions behind the program are described in Hanna (1989).

Figures 1 and 2 show results of running the program for different assumptions. All analyses shown are based on a 25 year old planning for the next 75 years, with an initial annual aftertax salary of \$31,000. She will remain single all her

life, and her real aftertax salary will increase at one percent per year until her retirement on her 65th birthday. The different patterns illustrated in the graphs are related to the value assumed for the elasticity of marginal utility with respect to consumption,  $-X$ . A value of  $X=1$  might be considered very thrifty, while a value of  $X=5$  might be representative of the typical American. Non-investment income increases to \$45,698 at age 64. Pensions other than from investments are \$22,849 per year after retirement. For the 'thrifty' consumer, the first year, optimal consumption is only \$19,570 (Figure 1) and 37% of income is saved (Figure 2). For the 'typical' consumer, nothing is saved the first year. The thrifty consumer saves early, and compound interest increases net worth enough to allow for dissaving by age 53. Real consumption is 2.6 times as high at age 64 as at age 25 for the thrifty consumer, but only 1.21 times as high at age 64 as at age 25 for the typical consumer.

REFERENCES

Hanna, S. (1989). Optimal life cycle savings. Proceedings of the Association for Financial Counseling and Planning Education.  
 Thaler, R. H. (1990). Anomalies: Saving, fungibility, and mental accounts. Journal of Economic Perspectives, 4, 193-205.  
 White, B. B. (1978). Empirical tests of the life cycle hypothesis. American Economic Review, 68, 547-560.



<sup>1</sup>Professor, Family Resource Management Department.